An Efficient Approximation for Minimum Latency Broadcast in Multi-Channel Multi-Hop Wireless Networks

Shan Shan, Wei Wang, Hongjie Du, Xiaofeng Gao, and Weili Wu

***Abstract*—In this paper, we discuss *the minimum latency broadcast problem* (MLB) *in multi-channel multi-hop wireless networks* (MLB-MC). This problem is *NP-hard* since its special version, MLB *in single-channel network* (MLB-SC) is proved to be *NP-hard* [1]. We design an efficient approximation for MLB-MC, analyze its approximation ratio, and evaluate its performance via numerical experiments. Furthermore, we give a general theorem as an upper bound to compare the performance between approximations for MLB-MC and MLB-SC.**

***Index Terms*—Wireless network, broadcast, approximation.**

I. INTRODUCTION

MULTI-HOP wireless networks, including wireless ad hoc networks and wireless sensor networks,etc. are decentralized networks in which all nodes cooperate together to fulfill a network task. To overcome low bandwidth constraint, multiple channels are facilitated for communication issues.

Broadcast is an effective data dissemination technique in wireless networks. Messages are spread from a source to the whole network through hop to hop transmissions. We assume one transmission takes one unit time, thus divide time into equal time slots. The minimum latency broadcast problem(MLB) is trying to find a interference-free broadcast schedule with minimum time slots. MLB is extensively studied in multi-hop wireless network [2] [3]. However, the usage of multi-channel brings new challenge to MLB. We name this optimization problem as MLB-MC. Correspondingly, MLB in single channel is MLB-SC. Multi-rate MLB and MLB-MC problems are considered in [4] [5].

In this paper, we consider MLB-MC with three types of interference, namely $collision$, $interference$ and $contention$. Fig.1(a) shows the influence of $collision$. If two nodes s_1, s_2 send messages to one destination r_1 in their transmission range R at the same time without detecting the existence of each other. Fig.1(b) illustrate $interference$. If receiver r_2 locates within the $interference\ range\ \alpha R$ of one sender s_1, it cannot receive message from other sender s_2 when s_1 is broadcasting. Fig.1(c) explains $contention$ which means a sender s_2 within the $contention\ range\ \beta R$ (also named as $carrier\ sensing\ range$) of another sender s_1 cannot broadcast while s_1 is broadcasting. Any type of interference happens, the transmission fails. Two transmission are $parallel\ transmission$ if none of the above happen. In this paper, we design an efficient approximation algorithm named k-coloring to solve MLB-MC. We provide its approximation ratio and prove its efficiency by numerical experiments. We also give a general theorem as an upper bound to compare approximations for MLB-MC and MLB-SC.

The rest of the paper is organized as follows. Section II gives the problem formulation. In Section III, we propose k-coloring with performance analysis and a universal theorem comparing algorithms for MLB-SC and MLB-MC. Simulation and conclusions are presented in Section IV and V.

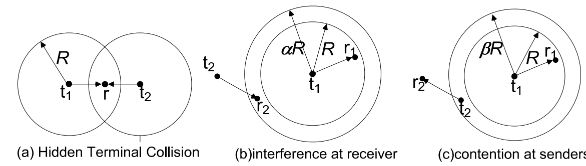

Fig. 1. 3 types of conflicts.

II. PROBLEM STATEMENT

We consider single source broadcast with K channels. For one broadcast process, each node in network should obey the broadcast rule which is each node should receive a copy before broadcasts and receive at least one copy of message from the source in the end. We use an undirected graph $G = (V, E)$ to represent our communication model, V and E are the vertex and edge set. We assume that all the nodes in the network have the equal transmission range R, interference range αR and carrier sensing range βR [2],$\beta \geq \alpha \geq 1$. $d(u, v)$ is Euclidean distance between node u and v, $u, v \in V$. Node u is a neighbor of node v only if $d(u, v) \leq R$, the neighbor set of node u is denoted as $N(u) = \{u \in V | (u, v) \in E\}$. Assume a non-overlapping orthogonal frequency channel set C, $|C| = K$. The channel node v uses to send messages is represented by $color_{send}(v)$ and the channel to receive is $color_{rec}(v)$, channel assignment to node v is denoted as $color(v)$.

Given broadcast source s, the hop counts from node v to s is the height of node v denoted as $h(v)$. To compute broadcast latency, we divide time into equal time slot unit, assume one transmission needs one time slot, node u transmits in the $t(u)$ time slot. So latency is measured by number of time slots each broadcast process needs, denoted as m. If two transmissions can be scheduled in the same time slot without any interference, they are parallel transmissions and only parallel transmissions can be scheduled in a same time slot. If two nodes u,v are parallel senders or parallel receivers, then the transmissions they involved are parallel transmissions. We define indicator functions $ps()$ and $pr()$ for parallel senders and receivers. If node u and v are parallel

Manuscript received January 13, 2010. The associate editor coordinating the review of this letter and approving it for publication was G.-H. Im.

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Digital Object Identifier 10.1109/LCOMM.2010.05.100070

senders, then $ps(u,v)=1$, else $ps(u,v)=0$. If node u and v are parallel receivers, then $pr(u,v)=1$, else $pr(u,v)=0$. If two receivers are parallel receivers, their senders are parallel senders.

Lemma 1. *Two nodes t_1, t_2 are parallel senders if $d(t_1,t_2) > max(\alpha+1,\beta)R$① $\vee$ $color_{send}(t_1) \neq color_{send}(t_2)$②. Two nodes are parallel receivers if $d(r_1,r_2) > (max(\alpha,\beta)+2)R$① $\vee$ $color_{rec}(r_1) \neq color_{rec}(r_2)$②.*

Proof: ① is proved in [2]. Obviously,in ② two transmissions in different channels are parallel transmissions. ■

Schedule problem is represented as an assignment of node transmissions to time slots and channels to nodes. Assign to each node transmission different channels to minimize the number of non-parallel transmissions. Assign to each node $u \in V$ a time slot $t(u)=i, 0 \leq i \leq m$, at which u transmit the message. If a node does not transmit, no time slot is assigned to it. Broadcast latency m is the number of time slots needed to finish the broadcast. *MLB-MC is formally defined as follows: Given $G=(V,E), s, R, \alpha, \beta, K, C,$ assign time slots and channels to each node u to get minimum latency m such that $\forall u, \forall v$,if $t(u)=t(v)=i$,$ps(u,v)=1$, $u,v \in V, 1 \leq i \leq m$.*

III. Approximation Algorithm and Analysis

A. Algorithm

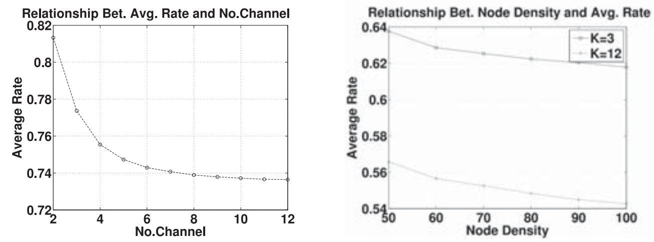
We name our approximation as k-coloring, which generalizes the algorithm in [2] for MLB-MC. The main idea is as follows: firstly a BFS tree T_{BFS} rooted at s is constructed, $T_{BFS} = T_1 \cup T_2 \cup T_3 ... T_h$, h is the height of T_{BFS}, $T_i = \{v \mid h(v)=i\}, 1 \leq i \leq h$. Then we form a global minimum weight maximum independent set(MIS) U, the weight $w_v = nc_v/IN_v$, nc_v is the set of nodes firstly covered by v, IN_v represents for the interference caused by adding v to U and is defined as $|s|$, $s=\{u \mid d(v,u) \leq max(max(\alpha+1,\beta)R, max(\alpha,\beta)+2)R) \wedge h(v)=h(u) \wedge u \in U\}$. The goal of constructing minimum weight global MIS instead of arbitrary global MIS is to eliminate the number of non-parallel transmissions. According to the depth of each node, $U = U_1 \cup U_2 ... U_h$. Next, we choose connectors to connect U into connected dominating set (CDS), connector set is denoted as UC. We select minimum connector set, because the smaller the size of CDS, the less number of non-parallel transmissions there are. Similarly, $UC = UC_1 \cup UC_2 ... UC_h$. Only nodes in $U \cup UC$ broadcast, broadcast is done from top to bottom. For each layer i, firstly, U_i receive from U_{i-1}, then U_i receive from UC_i. The main difference between k-coloring and [2] is the use of multiple channels, we assign different channels to non-parallel transmissions. An important concept used here is conflict graph $G_C=(V_C,E_C)$ [2] where V_C corresponds to the nodes to be scheduled, and there is an edge between two nodes iff. they cannot transmit message simultaneously. Conflict graph in which each node has radius $max(\alpha+1,\beta)R$ is sender conflict graph G_{C_c}; conflict graph in which each node has radius $(max(\alpha,\beta)+2)R$ is receiver conflict graph G_{C_r}. For each level i, we construct a receiver conflict graph $G_{C_{r_{u(i)}}}$ for U_i, then a sender conflict graph $G_{C_{c_{u(i)}}}$ for U_i, so each level i has two conflict graph. The last step is to schedule the broadcast, the idea which is similar to [2] is: for each level i, map the original problem to coloring algorithm on $G_{C_{c_{u(i)}}}$ and $G_{C_{r_{u(i)}}}$ and find the minimum number of colors m_{ci} and m_{ri} such that $G_{C_{c_{u(i)}}}$ and $G_{C_{r_{u(i)}}}$ are m_{ci}-colorable and m_{ri}-colorable separately. Time slots m_i needed for each level is $(m_{ci}+m_{ri})$. The broadcast latency m is the sum of m_i, $1 \leq i \leq h$. In the end, we present the channel assignment algorithm.

Input: U, UC, G_{C_t}, G_{C_r}
Output: $color_{send}(v), color_{rec}(v), v \in U \cup UC$
foreach $i, i \in 1, ...h$ **do**
  $S \leftarrow \emptyset$;
  foreach $u,\ u \in U_i$ **do**
    if $\exists np, np \in S,\ ps(np,u)=1$ **then**
      $color_{send}(u) \leftarrow color_{send}(np)$;
      $S \leftarrow S \bigcup \{u\}$;
    end
    if $\nexists np, np \in S,\ ps(np,u)=1$ **then**
      $color_{send}(u) \leftarrow l$, l is the least used channel in C;
    end
    foreach $n, n \in N(u) \cap UC_{i+1}$ **do**
      $color_{rec}(n) = color_{send}(u)$
    end
  end
  $S \leftarrow \emptyset$;
  foreach $u,\ u \in UC_i$ **do**
    if $\exists np, np \in S,\ ps(np,u)=1$ **then**
      $color_{send}(u) \leftarrow color_{send}(np)$;
      $S \leftarrow S \bigcup \{u\}$;
    end
    if $\exists np, np \in S,\ ps(np,u)=1$ **then**
      $color_{send}(u) \leftarrow l$, l is the least used channel in C;
    end
    foreach $n, n \in N(u) \cap U_i$ **do**
      $color_{rec}(n) = color_{send}(u)$
    end
  end
end

Algorithm 1: $K-coloring\ channel\ assignment$

B. Analysis

Generally, broadcast scheduling problem is mapped to graph coloring problem. A graph $G=(V,E)$ is said to be k-colorable, if there exists a mapping $f: V \to \{0,1,2,\cdots,k-1\}$ such that $f(u) \neq f(v)$ whenever vertices u and v are adjacent. The chromatic number $\chi(G)$ of graph G is defined to be the smallest k such that G is k-colorable. By Brooks's theorem, $\chi(G) \leq \Delta+1$, where Δ is the maximum degree of graph G. Given an arbitrary G, it is NP-complete to find $\chi(G)$. For MLB-SC, it is reduced to a coloring problem on the conflict graph $G_C=(V_C,E_C)$ and $m=\chi(G_C)$. For MLB-MC, it is reduced to Multichannel Coloring Problem(MCP). After the definition, we show that MCP can be reduced to the normal coloring problem and theorem2 shows the theoretical upper bound of MLB-MC problem.



(a) Vary No.Channel (b) Vary Node Density

Fig. 2. Relationship between density, channel and average rate.

Definition 1. *(Multichannel Coloring Problem, MCP) Assign a number from* $\{1, 2, \cdots, K\}$ *to each vertex of the conflict graph* G_C*, and partition* V_C *into* m *parts* $V_C(G) = \cup_{i=1}^{m} V_i$*, such that all nodes in* V_i *can send message simultaneously (i.e.,* $G_C[V_i]$ *is* m*-colorable). The objective is to minimize* $m =: \chi_K(G_C)$*.*

Theorem 2. *Let* $G_C = (V_C, E_C)$ *be the conflict graph. Then* $\chi_K(G_C) = \lceil \frac{\chi(G_C)}{K} \rceil$.

Proof: First, since G_C is $\chi(G_C)$-colorable, V_C can be partitioned into $\chi(G_C)$ parts $V_1, V_2, \cdots V_{\chi(G_C)}$ with V_i is an independent set for each i. Write $\chi(G_C) = Kq + r$ with $0 \le r < K$. Let

$\tilde{V_1} = V_1 \cup V_2 \cup \cdots \cup V_K$

$\cdots\cdots$

$\tilde{V_q} = V_{(q-1)K+1} \cup V_{(q-1)K+2} \cup \cdots \cup V_{qK}$,

$\tilde{V}_{q+1} = V_{qK+1} \cup V_{qK+2} \cup \cdots \cup V_{qK+r}$,

Then, nodes in $\tilde{V_i}$ can transmit message without interference (assign K channels to each independent set; each node in the same independent set acquires the same channel). It follows that $\chi_K(G_C) \le q + 1 = \lceil \frac{\chi(G_C)}{K} \rceil$.

On the other hand, suppose that V_C can be partitioned into s parts $V_1, V_2, \cdots, V_s$ such that each V_i is K-colorable. Assign each nodes in $V_1, V_2, \cdots, V_s$ a number (color) as follows:

V_1 : colored using colors in $\{1, 2, \cdots, K\}$;

V_2 : colored using colors in $\{K+1, K+2, \cdots, 2K\}$;

$\cdots\cdots$

V_s : colored using colors in $\{(s-1)K + 1, (s-1)K + 2, \cdots, sK\}$.

So $V_C = V_1 \cup V_2 \cdots \cup V_s$ is sK-colorable. By the definition of $\chi(G_C)$, we have $sK \ge \chi(G_C)$, which implies that $s \ge \lceil \frac{\chi(G_C)}{K} \rceil$. Hence $\chi_K(G_C) \ge \lceil \frac{\chi(G_C)}{K} \rceil$. Combining this with the inequality $\chi_K(G_C) \le \lceil \frac{\chi(G_C)}{K} \rceil$, the theorem follows. ■

Theorem 3. *Lower bound and upper bound of K-coloring Appro. ratio* ρ *are* $O(\frac{\max(\alpha^2, \beta^2)}{K})h$ *and* $O(\max(\alpha^2, \beta^2))h$*.*

Proof: First consider G_{C_c}, assume single channel case, $\forall i, 1 \le i \le h$,$\forall G_{C_{c_u(i)}}$, $\forall G_{C_{r_u(i)}}$, $v \in V_{C_{c_u(i)}} \cup V_{C_{r_u(i)}}$, $\max \delta(v)$ is a bounded constant. For node v, the number of neighbors in $G_{C_{c_u(i)}}$ or $G_{C_{r_u(i)}}$ can be obtained by computing the number of non-overlapping hexagons with radius $\frac{2}{\sqrt{3}} R$ [6] in the circle with radius $\max(\alpha + 1, \beta)R$.

$$\delta(v) = \frac{\pi(\max(\alpha + 1, \beta)R + \frac{R}{2})^2 - \frac{\sqrt{3}R^2}{2}}{\frac{\sqrt{3}R^2}{2}} = O(\max(\alpha, \beta)^2)$$

If G is a line graph, then bound of $\delta(v)$ can not be improved through assigning multiple channels, so lower bound of $\delta(v)$ is $O(\max(\alpha, \beta)^2)$. Upper bound of $\delta(v)$ has been proved to be $O(\frac{\max(\alpha, \beta)^2}{2})$ in theorem 2. Since $\forall i, 1 \le i \le h$, m_i is bounded, $m = \sum m_i$, so $\rho \in [O(\frac{\max(\alpha, \beta)^2}{K})h, O(\max(\alpha, \beta)^2)h]$. For G_{C_r}, it is the same. ■

IV. Simulation

In this section, we simulate K-coloring and CABS [2](Minimum Latency Broadcast Algorithm for single channel) using a simulator developed by ourself. In all experiments, nodes are randomly put in a square of 1000*1000, and the transmission range of each node is assumed to be 200,$\alpha = \beta = 2$. Each experiment is run 3000 times using random data generated from different random seeds and the average values are used to plot the figure. We measure the average rate(rate equals to latency in K-Coloring over CABS) by varying total number of channels K or node density. As we increase K or node density, average rate is decreasing which means performance of K-Coloring outperforms CABS.

V. Conclusion

In this paper, we studied the MLB-MC Problem and present an approximation algorithm with constant ratio. The simulation results shows the impact of utilizing multiple channels on the broadcast latency.

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